Analysis of a Causal Model of Crash Test Pulses

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Abstract

Crash testing in which a vehicle strikes a rigid barrier is common. However, published tests are usually limited to those performed for consumer programs or regulatory requirements, and crash performance at test velocities may not predict crash performance at other velocities. A differential equation model of vehicle behaviour can overcome some deficiencies of limited testing. This study examines the theoretical basis for a differential equation model of vehicle crash response. It describes the crash test data used to assess the validity of the differential equation, the criteria for selection of the data, and the simple results obtained when the differential equation model was applied to the data. Results are interpreted in terms of the theory, and implications and limitations of this interpretation are discussed. Conclusions are drawn, though these are tentative because the small sample size gave inconclusive results, and future work related to the study is suggested.

Introduction

Crash testing in which a vehicle strikes a rigid barrier is common, but published tests are usually limited to those performed for consumer programs or regulatory requirements; hence, the published crash test performance of a vehicle is generally limited to one or two tests encompassing a narrow range of impact velocities. However, crashes occur over a wider range of velocities than are found in published tests, and the crash performance of a vehicle in mandated test conditions may not predict crash performance at other speeds (Searson, Hutchinson, & Anderson, 2012).

Using a differential equation to model the vehicle crash pulse can overcome some limitations of testing at a limited number of impact velocities. The advantage of describing vehicle impact response using a differential equation is that it is causal; the state of the system can be extrapolated from the starting conditions (impact velocity in this case) and properties inherent to the system (structural properties of the vehicle and the impact barrier in this case). A causal model that uses a differential equation can therefore use results from published crash tests to predict the impact response of a vehicle at velocities not tested. The causal model may then be analysed using a large set of crash test data, such as the NHTSA crash test database, to determine whether it is compatible with the observed vehicle crash response observed in the published tests. If this analysis indicates that the model is compatible with the observed vehicle crash response, it might then be used to predict the vehicle crash response at speeds not tested, on the basis of published crash tests.

There are other reasons to be interested in developing causal models of crash test pulses; with the advent of primary safety technologies such as autonomous emergency braking, crash velocities may be modified, and hence assessment of the overall crash safety of vehicles will increasingly need to account for both secondary safety performance and active speed reductions. Developing equations relating crash test results and performance at velocities not tested may provide a basis for integrating the assessment of primary and secondary safety. The utility of causal models, in which vehicle performance is dependent on impact velocity has been recognised before, where they have been applied in pedestrian crash analysis (Edwards, Nathanson, & Wisch, 2014; Searson, Anderson, & Hutchinson, 2014; Hutchinson, Anderson, & Searson, 2012).

Causal models have also been developed to determine the vehicle occupant-restraint system response for a given input excitation (Huang, 2002). However, the input used in the Huang model is a “tipped equivalent” square wave or a sinusoidal excitation, calculated using information from an existing crash test. These inputs are limited to replicating the existing crash test impact velocity and vehicle response. Developing a causal model for the vehicle response at an arbitrary impact velocity would enable development of an entirely causal model which estimates occupant-restraint system dynamics for a given vehicle at velocities not tested, on the basis of existing test results.

This study first examines the theoretical basis for a differential equation model of vehicle crash response. It then describes the crash test data used to assess the validity of the differential equation, and the simple results that were obtained when the differential equation model was applied to this test data. These results are interpreted in terms of the theory, and the implications and limitations of this interpretation are discussed. Conclusions are drawn on the basis of this discussion, and possible future work related to the study is suggested.

Background

The majority of research into the relationship between vehicle crash pulse parameters and impact velocity has been undertaken in the context of vehicle crash reconstruction. Campbell analysed crash test data to determine relationships between impact velocity and residual crush (Campbell, 1974). This work and subsequent studies of the relationship between impact force and vehicle structural response form the basis of crash reconstruction computer programs (Strother, Woolley, James, & Warner, 1986). Jiang, Grzebieta, Rechnitzer, Richardson and Zhao (2003) examined a number of models used to relate vehicle crash performance to impact energy. The models were single linear equations as developed by (Campbell, 1974); bi-linear equations as described by Strother, Woolley, James and Warner (1986) and used by Varat, Husher and Kerkhoff (1994) and Neptune (1999), which account for non-linearity in the relationship between impact velocity and deformation; and constant-force equations as used by Wood and Mooney (1997) that approximate the vehicle structure as regions of constant-force deformation. Varat et al. (1994) also examined non-linearity in frontal crash structure response over a range of impact velocities and approximated the vehicle crash structure response with a quadratic equation for a number of vehicles.

The models relating impact velocity to crash performance that are used in crash reconstruction typically apply a linear, bi-linear, or quadratic relationship between impact velocity and vehicle residual crush, with an intercept calculated to optimise the fit of the linear relationship to test data (Sharma, Stern, Brophy, & Choi, 2007). There are however two important differences between models of the type developed by Campbell (1974) and the model analysed below. Unlike Campbell, dynamic crush was used instead of residual crush; Neptune (1999) suggested that dynamic crush is the more suitable measurement of crash severity, due to issues with the measurement of residual crush. McHenry and McHenry's (1997) work on the effects of crash structure restitution reinforces this approach, suggesting that restitution affects calculation of crash severity and that measurements recorded at the time of maximum dynamic crush are more suitable for calculating vehicle structural response. The model analysed below applies a causal physical model to this crash pulse parameter (dynamic crush), and uses this causal model to determine the relationship between impact velocity and the crash pulse parameter. Models which assume a linear relationship between impact velocity and residual crush also imply that the vehicle crash structure acts as a linear spring or dissipator; the model that is tested below allows for a non-linear force-deformation relationship for the vehicle crash structure. Similar to Campbell's model, the causal model analysed below is general, and may be applied to different crash configurations (e.g. offset tests, pole tests) if the appropriate data is available to generate the model constants.

Outside the field of crash reconstruction, studies have been conducted that reconstruct vehicle dynamics from acceleration data and assess injury risk as a function of impact velocity and other crash pulse parameters (Ydenius, 2010). But these studies have typically analysed real-world crash data (Stigson, Kullgren, & Rosén, 2012), and related the crash pulse parameters (mean acceleration, peak acceleration and velocity change) directly to occupant injury outcomes, rather than examining the intermediate results, in this case the change in vehicle response with changing impact velocity.

Theory

A differential equation relating acceleration (x''), instantaneous velocity (x') and instantaneous deformation (x) may be used to model a deforming vehicle crash test structure. This equation may take the form of Eq. (1):

$$mx'' - kx^n \left[1 + \left(\frac{b}{v_0}\right) x'\right] = 0 \quad (1)$$

The value of m, k, b and n are constant for a given vehicle; the value of n, the parameter that describes the non-linearity of the force-deformation function of the vehicle crash structure is investigated in this study. Eq. (1) is from Hutchinson (2013), developed from equations in Hunt and Crossley (1975) and Herbert and McWhannell (1977). Let the velocity sensitivity factor that relates force at a standard crush rate to force at a different crush rate be linearly dependent on the logarithm of the ratio of the two crush rates, as in Huang (2002, p. 314). Then, for a model with force proportional to x^n, Eq. (1) is compatible with the model, for values of n not too far from 1.

Eq. (1) contains a spring term kx^n; and a damping term $kx^n(b/v_0)x'$, and satisfies the condition that damping is zero for x = 0 and x' = 0, consistent with Hunt and Crossley (1975). Hutchinson (2013) used Eq. (1) to simulate the dynamics of a rigid body (a pedestrian head-form) impacting a deformable surface, and applied it in the context of a deformable object (the frontal crash structure of a vehicle) impacting a rigid barrier (Hutchinson, 2015).

From Eq. (1), a set of power functions, Eqs. (2-4) relating impact velocity (v_0) to dynamic crush (C, the maximum deformation experienced by the vehicle), peak acceleration (a_{peak}) and impact duration (t) may be derived, with exponents determined by the non-linearity coefficient n. Because restitution of a vehicle crash structure in a rigid barrier impact is small (Kerkhoff, Husher, Varat, Busenga, & Hamilton, 1993), Eq. (2) also applies to permanent crush, and Eq. (4) also applies to time of dynamic crush (t_m, the time at which the vehicle reaches maximum deformation).

$$C \propto {v_0}^{2/(n+1)} \quad (2)$$

$$a_{\text{peak}} \propto {v_0}^{2.n/(n+1)} \quad (3)$$

$$t \propto {v_0}^{(1-n)/(n+1)} \quad (4)$$

The value of n can be determined from existing crash pulses, by regressing known values of C, t_m and a_{peak} against v_0 for a range of impact velocities using Eqs. (2)-(4). If n values calculated using these equations are consistent, it would be evidence of the correctness of Eq. (1).

In this model, the value of n is greater than zero (Hutchinson, 2015). The exponent $2/(n+1)$ in Eq. (2) therefore varies between 0 and +2; the exponent $2.n/(n+1)$ in Eq. (3) varies between 0 and +2; and the exponent $(1-n)/(n+1)$ in Eq. (4) varies between -1 and +1. A value of n near 1, as for a linear spring, might be expected; models used in crash reconstruction assume a linear relationship between crush and the square root of crash energy (Sharma et al., 2007), and therefore a value of n equal to 1 for a constant vehicle mass.

Methods

Database

This study uses data sourced from the NHTSA Vehicle Crash Test Database (NHTSA, 2014). In 2014 when tests were selected for the present analysis, the database contained approximately 7500 instrumented crash tests, performed from 1965 onward. It is notable that in this database there are only a small number of vehicles that have been subjected to instrumented frontal crash tests over a range of velocities. When specific results in this study are discussed, 'vehicle' refers to the combination of a specific model year, make and model, for example a 2004 Honda Accord, and each vehicle in the sample set of tests selected from the database was tested at a number of velocities. Tests were chosen on the following basis:

- Vehicles were mid-size/full-size, transverse-front-engine, 4-door passenger cars, from model years 2004 to 2014. The vehicles were either front-wheel-drive or all-wheel-drive (based on a front-wheel-drive configuration).
- There were at least two full-frontal rigid-barrier crash tests for each vehicle conducted at significantly different impact velocities (11.1 m/s and 15.7 m/s).
- High-speed video of the crash test with timing information was available from the NHTSA database.

All tests in the NHTSA database which met the criteria above were included in this study. For six tests, displacement-time plots were available from the NHTSA database, and were used for validation of the calculated vehicle dynamic behaviour in these cases.

Crash Pulse Parameters for Assessment of Model

The values of C, t_m and a_{peak} may be calculated or determined directly using data from published crash tests. In addition, permanent crush (S) could be used in place of C in Eq. (2). Therefore, there are four candidate variables which may be used with Eqs. (2)-(4) to separately determine the value of n. An advantage of the differential equation is that it enables predictions to be made using the interconnectedness of the four crash pulse parameters; a consistent value of n calculated from observations of the different parameters provides evidence for the correctness of the differential equation.

However, there are potential issues using the parameters t_m, a_{peak} and S to assess the validity of Eq. (1). These issues are discussed in Appendix A. The value of n was therefore calculated on the basis of dynamic crush results. This does not allow the correctness of Eq. (1) to be assessed, but the sample size and impact velocity range of the available data mean that other parameters are not suitable to determine the value of n. Time of dynamic crush and peak acceleration results are included in Table A1 as they may be of general interest to the reader.

The results of the power-law regression used to calculate n were compared with linear regression on the same data, and the value of n was expected to determine the nature of the linear regressions between C and v_0. For example, if $n > 1$, the exponent $2/(n+1)$ in Eq. (2) is less than 1, a plot of C versus v_0 is concave downwards, and linear regression between C and v_0 is likely to have a positive intercept (C positive for $v_0 = 0$). If $n < 1$, $2/(n+1)$ in Eq. (2) is greater than 1, C versus v_0 is concave upwards, and linear regression between C and v_0 is likely to have a negative intercept (C negative for $v_0 = 0$).

Data Processing

1. Unfiltered B-pillar acceleration vectors were averaged to generate a ‘whole-vehicle’ acceleration vector for the undeformed portion of the vehicle. This was plotted against time, and the onset of acceleration was identified by examining the acceleration-time plot. The start of the crash pulse was defined as the time of acceleration onset, so as not to distort the results by including initial crush of a front bumper cover with very low stiffness. This was intended to ensure consistency in impact start times and dynamic crush measurements across vehicles with varying bumper cover designs. This differs slightly from the approach used for published results, which define impact start time based on a test signal. Calculated values for dynamic crush and time of dynamic crush may therefore vary slightly from published values.

2. The left B-pillar, right B-pillar and averaged acceleration vectors were zeroed, using data from the start of recording to 10 ms before the identified impact start time, so that there was no overall acceleration in the pre-impact interval.

3. The acceleration vectors were filtered at 300 Hz using an SAE J211-compatible second-order Butterworth filter, as described by Huang (2002, pp. 9-16). The acceleration vectors were integrated with respect to time using Simpson’s rule to generate velocity and displacement vectors.

4. To calculate peak acceleration values, the acceleration vectors were filtered at 100 Hz using the same SAE J211-compatible filter. The filtering eliminates short-lived acceleration spikes while preserving the whole-vehicle acceleration behaviour. Filtering allows comparison of peak acceleration values, as different tests were conducted at different sampling rates, and acceleration measurements from the unfiltered test data therefore have varying sensitivity to short-lived acceleration spikes.

5. Displacement of the right and left B-pillar were compared with each other; with published results when available (time-displacement data was published for 6 tests); and with high-speed video of the crash. High-speed video was compared with accelerometer data by determining B-pillar displacement of each side of the vehicle from the video at the start of impact and at maximum dynamic crush, and comparing the resulting change in displacement with the accelerometer-calculated value. The calculated displacement values were consistent each other, with video, and with published results. Maximum dynamic crush, peak acceleration and time of dynamic crush were then determined from the whole-vehicle displacement vector.

6. Tests were grouped by vehicle, and the natural logarithm of maximum dynamic crush $\ln(C)$ was plotted against the natural logarithm of impact velocity $\ln(v_0)$ for every test to determine the linear dependence of $\ln(C)$ on $\ln(v_0)$ for each vehicle. Regression, or direct calculation for vehicles with 2 tests, was used to estimate the exponent $2/(n+1)$, allowing n to be determined for each vehicle. To link these results to crash reconstruction methods, the linear dependence of C on v_0 was also briefly assessed.

7. ANCOVA testing was used to determine whether n values for different vehicles varied significantly, adjusting for impact velocity. For vehicles with four or more tests, the confidence interval for the relationship between $\ln(C)$ and $\ln(v_0)$ was calculated. The upper and lower confidence limits of the exponent $2/(n+1)$, and of n were then calculated.

8. An optimal common exponent for all vehicles in the study was determined by regression. When calculating the common exponent, the constant of proportionality implicit to Eq. (2) was allowed to vary for each different vehicle. A common value of n was then calculated.

9. ANCOVA testing was used to determine whether the n value for each vehicle varied significantly from the common n, adjusting for impact velocity. The sample data was separated into two groups for each vehicle; one group contained tests for the vehicle in question, while the common n value was calculated from the remaining tests in the sample. The point estimate value of n for these two groups were then compared to test the hypothesis that the n value for each individual vehicle was not equal to the common n value. It should be noted that this test has very low statistical power due to the small number of data-points available for each vehicle.

There are some variations between the dynamic crush results calculated in this analysis and the 6 published results in the NHTSA test database. This variation results from the impact start time being based on the onset of acceleration, rather than the test signal used to indicate the impact start time in published tests. Test 4776 exhibits the greatest variation in dynamic crush: there is a 51 mm difference in dynamic crush between published and calculated values, which corresponds to a 3 ms variation in impact start time.

Results

Crash Pulse Parameters

The first results of interest from analysis of the tests are the crash pulse parameters, derived from the vehicle acceleration during the barrier collision. These results are used to calculate the value of n and examine the validity of Eq. (1). Dynamic crush, peak acceleration and time of dynamic crush are summarised in Appendix A for the 41 tests of 15 vehicles. In the opinion of the authors, dynamic crush measurements are more suitable for determination of n than peak acceleration or time of dynamic crush. Peak acceleration and time of dynamic crush may be of general interest, but are not used to calculate n, for reasons discussed in Appendix A.

Dynamic crush at 11.1 m/s varied between 0.329 m and 0.539 m. At 13.4 m/s, dynamic crush ranged between 0.555 m and 0.605 m. At 15.7 m/s dynamic crush ranged between 0.589 m and 0.751 m. The Buick Lucerne 2006 exhibited the highest dynamic crush at 11.1 m/s and 15.7 m/s. Peak acceleration at 11.1 m/s varied between 203 m/s^2 and 296 m/s^2. At 13.4 m/s, peak acceleration varied between 276 m/s^2 and 315 m/s^2. At 15.7 m/s, peak acceleration varied between 262 m/s^2 and 509 m/s^2.

Time of dynamic crush at 11.1 m/s ranged between 55 ms and 96 ms. At 13.4 m/s, time of dynamic crush ranged between 0.070 s and 0.081 s. At 15.7 m/s, time of dynamic crush ranged between 0.064 s and 0.084 s. The Buick Lucerne 2006 has the longest time of dynamic crush at 11.1 m/s and at 15.7 m/s. For all vehicles, time of dynamic crush was approximately constant across the tested range of impact velocities.

Dependence of Dynamic Crush on Impact Speed

The value of the exponent for dynamic crush $2/(n+1)$ and the associated value of n were calculated for each vehicle based on the calculated dynamic crush results, and are summarised in Table 1. The values of n were between 0.08 (Suzuki Kizashi 2010) and 1.54 (Toyota Camry 2011).

For vehicles with four or more tests (Ford Taurus 2010-11, Honda Accord 2004, Hyundai Sonata 2011, Toyota Camry 2004), the 95 percent confidence interval for the exponent $2/(n+1)$ and the corresponding interval for n were calculated. In the first 3 cases, the confidence intervals for n were (0.40, 2.24), (0.42, 1.18), and (0.06, 3.84) - these confidence intervals are too wide to be useful. For the 2004 Toyota Camry, the 95 percent confidence interval was (-0.04, 2.19) – this confidence interval provides no information, as the exponent must be between 0 and 2 for a positive value of n.

The variation in the point estimates for the value of n of different vehicles may indicate that n varies depending on the vehicle. Using ANCOVA to test the hypothesis that n values are different for different vehicles gives a p-value of 0.08 when adjusted for impact velocity, failing to reject the null hypothesis that the values of n are the same for different vehicles. The near-significance of this result suggests that the value of n may vary for different vehicles, but the statistical power of the test is very low. Confidence intervals for values of n for individual vehicles are also large, which may indicate that variation in n for different vehicles is the result of random error in the results, coupled with the very small sample size for each vehicle.

The linear dependence of C on v_0 was also fitted to test data, and is summarised in Table 2. As would be expected from the curvature of power functions, the intercept (the value of C at $v_0 = 0$) was positive for vehicles for which the exponent $2/(n+1)$ was less than about 1, and negative for vehicles for which the exponent was greater than about 1.

Table 1: Calculated values of the two relationships fitted to the dynamic crush and impact velocity data: the power-law exponent and resulting values of n from a fitted power-law curve of C (m) vs v_0 (m/s); and the fitted linear dependence of C (m) on v_0 (m/s)

Vehicle	Exponent	n	Linear Intercept	Linear Slope
Honda Accord 2004	1.16	0.72	-0.087	0.050
Honda Accord 2008	1.46	0.37	-0.261	0.064
Toyota Camry 2004	1.07	0.86	-0.015	0.043
Toyota Camry 2007	0.89	1.25	0.058	0.036
Toyota Camry 2011	0.79	1.54	0.115	0.033
Ford Taurus 2004	1.34	0.49	-0.193	0.058
Ford Taurus 2010	1.03	0.94	-0.014	0.042
Mitsubishi Galant 2004	1.04	0.93	0.002	0.040
Chevrolet Malibu 2008	0.83	1.42	0.096	0.035
Ford Fusion 2006	0.88	1.27	0.069	0.039
Hyundai Sonata 2006	0.81	1.48	0.098	0.031
Hyundai Sonata 2011	1.15	0.74	-0.075	0.044
Nissan Maxima 2009	0.82	1.43	0.094	0.034
Suzuki Kizashi 2010	1.85	0.08	-0.383	0.064
Buick Lucerne 2006	0.95	1.11	0.033	0.046

Calculation of Common n Value

The wide confidence intervals around n values for individual vehicles may indicate that there is insufficient data to reliably calculate n for individual vehicles. Calculation of a common value of n across all vehicles may therefore provide a more reliable result. The approach of grouping vehicles by type (car, van, truck), then by engine configuration (orientation, capacity, number of cylinders) has been used in Jiang et al. (2003); the approach was applied to a set of vehicle tests to determine the appropriate model (linear, bi-linear or quadratic) for the vehicle configuration being assessed. A common n across all vehicles in the sample was calculated by performing linear regression on the log-transformed v_0 and C for all tests in the sample, treating the vehicle make/model/year as an independent variable. Calculation of a common exponent $2/(n+1)$ and n yielded an exponent of 1.08, with a confidence interval of (0.95, 1.20), corresponding to n = 0.85, with a confidence interval of (0.66, 1.10).

Using ANCOVA to test the hypothesis that n for each vehicle was not equal to the common n indicated that in most cases the point estimate of n for an individual vehicle did not differ significantly from the common n. In the case of the 2010 Suzuki Kizashi, the n value was significantly different to the common n value, with a p-value of 0.007, adjusted for impact velocity. The difference between the Suzuki Kizashi result, with a calculated n of 0.08, and the common n of 0.85 may indicate that the variation in n values precludes the use of a common n. The n values for other vehicles in the sample did not vary significantly from the common n; p-values for the hypothesis that the common n differed from the individual vehicle n ranged between 0.271 and 0.986, failing to reject the null hypothesis that the values of n for those vehicles are the same as the common n.

Discussion

The common n derived from aggregated data for dynamic crush (n = 0.85) appears compatible with the crash test data used in this analysis. Variation in n for different vehicles is marginally significant in ANCOVA testing, but the statistical power of these tests is low. The n values for individual vehicles in the study range between 0.08 and 1.54, but due to the large confidence intervals around these estimates, only one vehicle, the 2010 Suzuki Kizashi for which n = 0.08, varies significantly from n = 0.85. For individual vehicles, the linear dependence results between impact velocity and dynamic crush, and between impact velocity and peak acceleration, are consistent with n around 0.85. The approximately constant time of dynamic crush for individual vehicles across a range of impact velocities is consistent with this value of n.

The small sample size used for this analysis means that the results were not conclusive. The inclusion criteria for this study limited the sample to 41 tests conducted over a small range of impact velocities, in which the maximum v_0 was approximately 140 percent of the minimum v_0. The limited dataset causes the analysis to be sensitive to random variation in test results, and results in wide confidence intervals for the n values calculated for individual vehicles. Because the confidence intervals are so large, it is uncertain whether the n values for the different sample vehicles are incompatible with one another. The small speed range and small dataset also limit the parameters that can be analysed to test Eq. (1). Values of n calculated from peak acceleration measurements are affected by the sensitivity of a_{peak} to filtering; and n values calculated from t_m are sensitive to small random errors in time of dynamic crush measurements around n = 1 when the range of test impact velocities is small.

At present, the analysis has only been applied to full-width rigid-barrier frontal impacts. The frontal impact that was analysed in the present study was chosen primarily because the impact configuration is used over a range of impact velocities, allowing the change in crash pulse parameters with impact velocity to be calculated. However, the theoretical basis of the analysis does not preclude its application to other impact configurations; spring and damping terms similar to those calculated from the present application may be calculated from other crash types, such as moderate overlap or small overlap impacts. In these other types of impact, the dynamic crush of the affected portion of the vehicle crash structure would be more difficult to determine, because the whole vehicle does not reach zero velocity during the crash. This might be resolved by calculating dynamic crush based on the x-acceleration of the affected side of the vehicle; for example, if the left side of the vehicle impacts the barrier, displacement could be calculated by integrating the x-acceleration measured at the left B-pillar. However, these values of n are expected to be different to the n calculated for a full-width rigid barrier crash test, and the lack of suitable data at a range of impact velocities mean that it is not possible to calculate n using the crash test data that was available for this analysis.

Further crash testing undertaken over a wider range of impact velocities would be required to produce a dataset suitable for further analysis. A larger dataset covering a wider range of impact velocities would reduce the influence of random errors on the calculation of n from time of dynamic crush. This could in turn provide evidence to assess the correctness of Eq. (1), by allowing comparison of n values calculated from Eqs. (2) and (4). Testing the consistency of the model for a single vehicle could be undertaken with three or four tests over a larger range of impact velocities (for example between 10 m/s and 20 m/s).

To determine whether there is a consistent value of n for vehicles more generally would require similar testing of a range of vehicles. If the value of n varied significantly for each different vehicle make/model, the practicality of the method and the advantages in comparison to physical testing are minimal. However, if the model is valid and a consistent n is found for all vehicles or for a given vehicle configuration and impact type, it allows estimation of the vehicle crash pulse parameters on the basis of one test. A consistent value of n combined with the differential equation therefore enables crash pulse (duration, peak acceleration, and shape) to be estimated at speeds not tested. However, it is unlikely that existing crash testing programs would change to require crash tests over a wide range of impact velocities undertaken on a regular basis.

While it may be impractical to test every vehicle make/model enough times to eliminate the uncertainty in the model, it may be useful to use the model to compare crash test data with theory. The model detailed herein may provide a method of understanding how crash test results change at different impact velocities, or help to draw attention to unexpected crash test results or problems with crash test data.

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Appendix A – Choice of crash pulse parameters for model analysis

There are four crash pulse parameters that may be used to calculate the value of n; dynamic crush, time of dynamic crush, permanent crush and peak acceleration. These parameters may be derived from the vehicle crash pulse, or recorded after the crash test; peak acceleration may be read directly from the crash pulse, dynamic crush and time of dynamic crush can be calculated through integration of the crash pulse, and permanent crush can be measured directly from the vehicle. However, three of these parameters (peak acceleration, time of dynamic crush and permanent crush) have issues which make them unsuitable for testing of the differential equation with the available crash test data. These issues are outlined below:

- Values of n predicted using a_{peak} may differ from those predicted using C and t_m, because inhomogeneity of the frontal crash structure may lead to multiple acceleration peaks over the crash pulse, while C and t_m are integrated quantities that are calculated from the whole crash pulse (Hutchinson, 2015). The inhomogeneity in acceleration-time behaviour makes a_{peak} sensitive to filtering frequency, because filtering will attenuate short-duration acceleration peaks more than long-duration peaks. If the duration of a_{peak} varies for different crash tests, the attenuation of a_{peak} caused by filtering at one frequency (Society of Automotive Engineers, 1995) may vary for different crash tests, affecting the calculation of n.

- The value of t_m is insensitive to variation in v_0 for values of n near 1 (the value used in crash reconstruction). Taking n = 1 as a starting estimate, consider a range of n values between 0.5 and 1.5. This range is chosen because Hutchinson (2015) found that based on published crash test data, n might be taken as about 0.5 for front-wheel-drive cars (crash test results for rear-wheel-drive cars were also examined in that study). Selecting a range of n = 0.5 to n = 1.5 extends equally either side of n = 1 and includes Hutchinson's n = 0.5. For n between 0.5 and 1.5, the exponent $(1 - n)/(n + 1)$ will change between 0.33 and -0.2. As the lowest impact velocity is 70 percent of the highest impact velocity in the data used for this study, a change in the exponent from 0.33 to -0.2 would result in a 9 percent variation in time of dynamic crush. Therefore, it might be expected that small random errors in t_m may lead to large changes in the inferred value of n, making t_m ill-suited to the determination of n while using the current set of tests, as they were conducted over a small range of impact velocities.

- Permanent crush is omitted from Table A1 and disregarded as a parameter for assessment of the differential equation model due to uncertainties in its measurement. The NHTSA permanent crush measurement method does not disregard the gap between the deformed vehicle structure and the bumper cover. As a result, the bumper cover may rebound while the underlying structure remains crushed (Neptune, 1999), resulting in under-prediction of permanent crush. The resulting value of n may therefore be inconsistent with that found for dynamic crush even if Eq. (1) is applicable.

For these reasons, peak acceleration, time of dynamic crush and permanent crush are set aside for the analysis undertaken here, and peak dynamic crush is chosen as the measurement from which $\boldsymbol{n}$ is calculated. Peak acceleration and time of dynamic crush are included in Table A1 below for the general interest of the reader.

Table A1: Summary of crash pulse parameters for each vehicle test; included below are impact velocity (v_0), dynamic crush (C), peak acceleration (a_{peak}) and time of dynamic crush (t_m).

Vehicle	NHTSA Test	v_0 (m/s)	C (m)	a_{peak} (m/s^2)	t_m (s)
Honda Accord 2004	5104	11.06	0.456	227	0.078
Honda Accord 2004	5215	13.36	0.586	263	0.081
Honda Accord 2004	5145	15.69	0.685	339	0.076
Honda Accord 2004	5139	15.70	0.708	317	0.077
Honda Accord 2004	5062	15.72	0.677	348	0.074
Honda Accord 2008	6481	11.06	0.442	296	0.069
Honda Accord 2008	6229	15.64	0.733	328	0.080
Toyota Camry 2004	5071	11.00	0.437	284	0.066
Toyota Camry 2004	5216	13.44	0.605	294	0.076
Toyota Camry 2004	5138	15.64	0.675	380	0.075
Toyota Camry 2004	4871	15.78	0.625	405	0.070
Toyota Camry 2007	6152	11.06	0.457	269	0.068
Toyota Camry 2007	5675	15.69	0.624	328	0.069
Toyota Camry 2011	7793	11.00	0.472	294	0.073
Toyota Camry 2011	6953	15.56	0.620	373	0.067
Ford Taurus 2004	4987	11.06	0.446	220	0.071
Ford Taurus 2004	5143	15.53	0.698	262	0.080
Ford Taurus 2004	4776	15.69	0.719	289	0.081
Ford Taurus 2010	7302	11.06	0.445	293	0.067
Ford Taurus 2010	7271	11.06	0.449	275	0.068
Ford Taurus 2010	6964	15.69	0.619	457	0.066
Ford Taurus 2010	6808	15.69	0.661	398	0.071
Mitsubishi Galant 2004	5238	11.06	0.427	287	0.065
Mitsubishi Galant 2004	5164	13.22	0.555	327	0.070
Mitsubishi Galant 2004	4866	15.50	0.604	380	0.065
Chevrolet Malibu 2008	6268	15.59	0.640	325	0.069
Chevrolet Malibu 2008	6448	11.06	0.482	259	0.071
Ford Fusion 2006	5546	15.69	0.685	305	0.076
Ford Fusion 2006	5821	11.06	0.503	203	0.078
Hyundai Sonata 2006	5453	15.74	0.591	509	0.064
Hyundai Sonata 2006	5730	11.06	0.444	281	0.067
Hyundai Sonata 2011	6940	15.64	0.604	390	0.067
Hyundai Sonata 2011	7002	15.64	0.589	372	0.067
Hyundai Sonata 2011	7203	15.69	0.653	417	0.074
Hyundai Sonata 2011	7792	11.06	0.413	271	0.065
Nissan Maxima 2009	6462	15.56	0.614	408	0.065
Nissan Maxima 2009	7046	11.06	0.464	254	0.071
Suzuki Kizashi 2010	6858	15.69	0.628	384	0.072
Suzuki Kizashi 2010	7009	11.06	0.329	263	0.055
Buick Lucerne 2006	5589	15.68	0.751	298	0.084
Buick Lucerne 2006	5741	11.06	0.539	224	0.096