A Comparison of Statistical Methods in Interrupted Time Series Analysis to Estimate an Intervention Effect

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Co-authors

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Outline

• Mandatory Helmet Legislation in NSW

• Interrupted Time Series Analysis

• Full Bayesian and Empirical Bayes methods

• Conclusions
Mandatory Helmet Law (MHL) in NSW

• Applies to all age groups

• Came into effect in two stages
  ➢ Adults (>16): January 1, 1991
  ➢ Children: July 1, 1991

• MHL led to much greater helmet wearing rate (>80% post law)

• Associated with fewer bicycle related head injuries
  ➢ Solid evidence for helmet wearing in lowering bicycle related head injuries from biomechanical and epidemiological studies
A Simple Analysis

• Comparing single pre- and post-intervention does not give the full picture:
  
  ➢ Ignores any trends before and after intervention
  
  ➢ Ignores any cyclical patterns
  
  ➢ Variances around the mean before and after intervention may be different
  
  ➢ Intervention may be immediate or delayed
  
  ➢ Doesn’t take into account any possible autocorrelation
Interrupted Time Series (ITS)

- A type of time series where we know the specific point at which an intervention (interruption) occurred
  - No randomisation
  - Definitive pre- and post-intervention periods

- Type of quasi-experimental design
  - ‘Strongest, quasi-experimental approach for evaluating longitudinal effects of intervention’ (Wagner et al. 2002)

- Important comparisons can be made between pre- and post intervention
  - Change in level (immediate effect)
  - Change in slope (gradual effect)
A Basic Model

- Using segmented regression (Wagner et al., 2002),

\[ Y_t = \beta_0 + \beta_1 \times T_t + \beta_2 \times I + \beta_3 \times T_t \times I + \epsilon_t \]

Where
- \( Y_t \) is the outcome of interest
- \( T_t = ..., -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, ... \) (intervention occurs when \( T_t = 0 \))
- \( I \) is a dummy variable
- \( \epsilon_t \) is the random error

RSRPE Conference, 2013
A Basic Model (cont’d)

• Using segmented regression (Wagner et al., 2002),

\[ Y_t = \beta_0 + \beta_1 \times T_t + \beta_2 \times I + \beta_3 \times T_t \times I + \varepsilon_t \]

Where
- \( \beta_0 \) estimates the base level of the outcome
- \( \beta_1 \) estimates the base trend of the outcome (change with time in pre-intervention period)
- \( \beta_2 \) estimates the change in level in the post-intervention period (\( H_0: \beta_2 = 0 \))
- \( \beta_3 \) estimates the change in trend in the post-intervention period (\( H_0: \beta_3 = 0 \))
Threats to Internal Validity

• Factors other than the intervention may influence the dependent variable

• For instance, changes in head injury rate may due to
  - Decline in the number of cyclists
  - Construction of cycling infrastructure

• Use of a dependent, non-equivalent, no-intervention control group to account for unmeasured confounding
  - We use arm injury

• More discussion found in Olivier et al. (2013)
Model Specification for Injury Counts

- Log-linear negative binomial regression model expressed using Poisson-Gamma mixture

\[ Y_i \mid \eta_i \sim \text{Poisson} \left( \eta_i \mu_i \right) \]
\[ \eta_i \sim \text{Gamma}(\alpha, \alpha) \]
\[ \log(\mu_i) = \beta_0 + \beta_1 \text{TIME} + \beta_2 \text{INJURY} + \beta_3 \text{LAW} + \beta_4 \text{TIME} \times \text{INJURY} + \beta_5 \text{TIME} \times \text{LAW} + \beta_6 \text{INJURY} \times \text{LAW} + \beta_7 \text{TIME} \times \text{INJURY} \times \text{LAW} + \log(\text{exposure}), \] (1)

where
- \text{TIME} = -17.5, \ldots, -0.5, 0.5, \ldots, 17.5
- \text{INJURY} is a dummy variable = \begin{cases} 0 & \text{arm injury} \\ 1 & \text{head injury} \end{cases}
- \text{LAW} is a dummy variable = \begin{cases} 0 & \text{pre-MHL} \\ 1 & \text{post-MHL} \end{cases}
- NSW population size is used as exposure
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where

- \( \beta_6 \): any differential changes in head injuries as compared to arm injuries from pre- to post MHL
- \( \beta_7 \): any differences in the rate of change of head and arm injuries between pre- and post MHL
Full Bayesian Method

• The above model can be estimated using maximum likelihood approach in SAS (Walter et al., 2011)

• **Full Bayesian (FB) method** as a powerful alternative
  - Combining likelihood and prior belief to generate posterior distribution of unknown parameters
  - A non-informative prior distribution is adopted in the absence of specific prior information

• Advantages
  1. Allow estimation of models with smaller sample sizes since Bayesian methods do not depend on their asymptotic properties
  2. Ability to include prior knowledge on parameter values into the model
  3. Enables one to implement very complex hierarchical models where the likelihood function is intractable
Full Bayesian Model Estimation

- Disadvantages
  1. Posterior distributions only analytically tractable for only a small number of simple models $\rightarrow$ simulation-based Markov chain Monte Carlo (MCMC) methods
  2. MCMC methods can be computationally intensive

- We will use MCMC techniques, in particular, Gibbs sampling, by implementing the model in the WinBUGS package

- The Gibbs sampler generates a sample from the joint posterior distribution by iteratively sampling from each of the univariate full conditional distributions

- For our model (1), we assign the following non-informative prior distributions:
  - Regression coefficients: $\beta_i \sim \text{Normal} (0, 1000), \quad i = 0, 1, ..., 7$
  - Dispersion parameter: $\alpha \sim \text{Uniform} (0.5, 200)$
Empirical Bayes Method

- Related to the FB method in combining current data with prior information to obtain an estimate
- The parameters of the prior distribution are estimated from existing data and then used assuming there is no uncertainty
- Extensively used in the analysis of traffic safety data, particularly for before-after evaluation of road safety treatments; need to evaluate Safety Performance Functions

**Advantage:**
- Easy to implement
- Less computationally costly than FB

**Disadvantages:**
- Do not fully account for all uncertainties as in FB
- May result in unrealistically small standard errors
Empirical Bayes Method

- We apply a particular EB procedure by French and Heagerty (2008)

- Fit a regression model to data prior to policy intervention and use the model to form a trajectory of outcomes in periods after intervention

\[ Y_i \mid \eta_i \sim \text{Poisson} (\eta_i, \mu_i) \]
\[ \eta_i \sim \text{Gamma}(\alpha, \alpha) \]
\[ \log(\mu_i) = \delta_0 + \delta_1 \text{TIME}_{\text{PRE}} + \delta_2 \text{INJURY} + \delta_3 \text{TIME}_{\text{PRE}} \times \text{INJURY} + \log(\text{exposure}) \]

- Contrast post-intervention observations with their expected outcomes under the absence of a policy intervention

\[ \Delta_i = \log(Y_i) - \log(\hat{\mu}_i) \]
Empirical Bayes Method

- We then model this contrast to obtain estimates for a policy effect and assess its significance by using standard statistical test:

\[ \Delta = \delta_4 + \delta_5 \text{TIME}_\text{POST} + \delta_6 \text{INJURY} + \delta_7 \text{TIME}_\text{POST} \times \text{INJURY} + \varepsilon \]

- \( \delta_4 \) and \( \delta_5 \) : baseline level and slope change for arm injuries after intervention respectively
- \( \delta_6 \) and \( \delta_7 \) : differential level and slope changes in head and arm injuries post-law respectively
## Results

**Table 1. Negative binomial model estimates using MLE, FB and EB methods**

<table>
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<tr>
<th>Variable</th>
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<td>Intercept ($\beta_0$ or $\delta_0$)</td>
<td>-11.470 (-11.613,-11.326)</td>
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<tr>
<td>TIME ($\beta_1$ or $\delta_1$)</td>
<td>-0.005 (-0.019,0.009)</td>
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<td>INJURY ($\beta_2$ or $\delta_2$)</td>
<td>0.072 (-0.128,0.272)</td>
<td>0.071 (-0.136,0.287)</td>
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<td>LAW ($\beta_3$ or $\delta_4$)</td>
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Negative estimate of β₃ and δ₄ indicate overall injury counts decreased after the law.
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Significant and negative estimate of $\beta_6$ and $\delta_6$: head injuries dropped by more than arm injuries post-law.
Results

Figure 1. Head vs. arm injury counts and fitted model (FB) for 18 months prior and post MHL
Empirical Bayes Results

- Parameter estimates similar to MLE and FB
- Standard errors are not directly comparable
- Mean contrasts for arm injuries does not significantly differ from zero ($\hat{\delta}_4 = -0.145$, s.e. = 0.086, $p$ value = 0.102)

- Mean contrasts for head injuries is significantly different from zero ($\hat{\delta}_4 + \hat{\delta}_6 = -0.447$, s.e. = 0.086, $p$ value = $1.14 \times 10^{-5}$)

- Significant difference between head and arm contrasts ($\hat{\delta}_6 = -0.302$, s.e. = 0.122, $p$ = 0.019)
Results

Figure 2. Head and arm injury counts with pre-policy estimation (solid line) and post-policy prediction (dashed line)
Summary of Analysis

• Three estimation methods give similar results
• Statistically significant drop in cycling head injuries after MHL
• Estimated legislation attributable drop in head injuries are 27.6% and 26.1% using FB and EB methods, comparable to 27.5% in the study by Walter et al. (2011)
• Comparing to frequentist maximum likelihood and EB approaches, the FB method
  - Better accounts for uncertainty in the sample
  - Models negative binomial distribution as a hierarchical Poisson-Gamma mixture distribution; allows other distributions (eg. Poisson-Lognormal) to be implemented
  - May be computationally costly and may have convergence issues

Conclusions
References


Thank you!